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# FACTORIZATION EFFECTS IN A MODEL OF UNSTABLE PARTICLES

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The effects of factorization are considered within the framework of the model of un-stable particles with a smeared mass. It is shown that two-particle cross section and three-particle decay width can be described by the universal factorized formulae for an unstable particles of an arbitrary spin in an intermediate state. The exact factorization is caused by the specific structure of the model unstable-particle propagators. This result is generalized to complicated scattering and decay-chain processes with unstable particles in intermediate states. We analyze applicability of the method and evaluate its accuracy.

*Keywords*: factorization; unstable particles

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## 1. Introduction

Unstable particles (UP's) are described usually by dressed propagator or with the help of the $S$-matrix with complex pole. The problems of these descriptions have been under considerable discussion for many decades [1]–[7] (and references therein). There are also other approaches such as time-asymmetric QFT of UP's [6], effective theory [8, 9], modified perturbative theory approach [10], and phenomenological QF model of UP's with smeared mass [11, 12]. In this work we consider some remarkable properties of the model [11, 12] which are caused by mass smearing and lead to the factorization effects in the description of the processes with UP in an intermediate state.

The model under consideration is based on the time-energy uncertainty relation (UR). Despite their formal uniformity, various UR's have different physical nature. This point has been discussed for many years beginning with Heisenberg's formu-lating the uncertainty principle (for instance, see [13]–[16] and references therein).

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The first model of UP based on the time-energy UR was suggested in [17]. Time-dependent wave function of UP in its rest frame was written in terms of the Fourier transform which may be interpreted as a distribution of mass values, with a spread, $\delta m$, related to the mean lifetime $\delta\tau = 1/\Gamma$ by uncertainty relation [14, 17]:

$$\delta m \cdot \delta\tau \sim 1, \ \text{ or } \ \delta m \sim \Gamma \ (c = \hbar = 1). \tag{1}$$

Thus, from the time-energy UR for the unstable quantum system, we are led to the concept of mass smearing for UP, which is described by UR (1). Implicitly (indirectly) the time-energy UR, or instability, is usually taken into account by using the complex pole in $S$-matrix or dressed propagator which describes UP in an intermediate state. Explicit account of the relation (1) is taken by describing UP in a final or initial states with the help of the mass-smearing effect. From Eq. (1) it follows that this effect is noticeable if UP has a large width. Mass smearing was considered in the various fields of particle physics—in the decay processes of UP with large width [12], in the boson-pair production [18, 19], and in the phenomenon of neutrino oscillations [16, 20]. In these papers, the efficiency of the mass-smearing conception was demonstrated in a wide class of processes.

The effects of exact factorization in the two-particle scattering and three-particle decay were shown for the cases of scalar, vector, and spinor UP in Refs. [21, 22]. Factorized formulae for cross section and decay width were derived exactly (without any approximations) for the tree-level processes in the physical gauge. However, the results can be generalized taking account of principal part of radiative corrections [12, 18, 19, 21] (see Section 3). The factorization method that is based on the exact factorization in the simplest processes was suggested in [23]. In this work we generalize the results of [23] to the case of UP with spin $J = 3/2$ and apply them to some complicated processes. In the second section we describe the main elements of the model that lead to the effects of factorization. These effects in the case of the simplest processes are considered in Section 3 for UP with $J = 0, 1/2, 1, 3/2$. The factorization method based on the results of the third section is applied to some more complicated processes in Section 4. In this section, we also consider the accuracy of the calculations. Some conclusions are made concerning the applicability and advantages of the method in the fifth section.

## 2. Model of UP with a smeared mass

In this section we present the elements of the model [11, 12] that are used directly in the factorization method. The field function of the UP is a continuous superposition of the standard ones defined at a fixed mass with a weight function of the mass parameter $\omega(\mu)$ which describes mass smearing. As a result of this, an amplitude of the process with UP in a final or initial state has the form:

$$A(k, \mu) = \omega(\mu)A^{st}(k, \mu), \tag{2}$$

where $A^{st}(k, \mu)$ is an amplitude defined in a standard way at fixed mass parameter $\mu$, and $\omega(\mu)$ is a model weight function.

The model Green's function has a convolution form with respect to the mass parameter $\mu$ (the Lehmann representation). In the case of scalar UP it is as follows:

$$D(x) = \int D(x, \mu)\, \rho(\mu)\, d\mu, \ \ \rho(\mu) = |\omega(\mu)|^2, \tag{3}$$

where $D(x, \mu)$ is defined in a standard way for a fixed $\mu = m^2$ and $\rho(\mu)$ is a probability density of the mass parameter $\mu$.

The model propagators of scalar, vector, and spinor unstable fields in momentum representation are given by the following expressions [12]:

$$D(q) = i \int \frac{\rho(\mu)\, d\mu}{q^2 - \mu + i\epsilon}, \ \ D_{mn}(q) = -i \int \frac{g_{mn} - q_m q_n/\mu}{q^2 - \mu + i\epsilon} \rho(\mu)\, d\mu,$$

$$\hat{G} = i \int \frac{\hat{q} + \sqrt{\mu}}{q^2 - \mu + i\epsilon} \rho(\mu)\, d\mu, \qquad q = \sqrt{(q_i q^i)}. \tag{4}$$

The model propagators are completely defined if the function $\rho(\mu)$ is determined.

Here, we generalize the results of the works [12, 21, 22] to include the case of unstable fields with spin $J = 3/2$. The propagator of this field is defined in [24, 25] and smearing its mass, $M^2 \to \mu$, gives:

$$\hat{G}_{mn}(q) = \int \rho(\mu)\, d\mu \{ -\frac{\hat{q} + \sqrt{\mu}}{q^2 - \mu + i\epsilon} (g_{mn} - \frac{1}{3}\gamma_m \gamma_n - \frac{\gamma_m q_n - \gamma_n q_m}{3\sqrt{\mu}} - \frac{2}{3}\frac{q_m q_n}{\mu}) \}. \tag{5}$$

Determination of the weight function $\omega(\mu)$ or corresponding probability density $\rho(\mu) = |\omega(\mu)|^2$ can be done with the help of the various methods [12]. Here we consider the definition of $\rho(\mu)$ which leads to the effect of exact factorization. We match the model propagator of scalar UP to the standard dressed one:

$$\int \frac{\rho(\mu)d\mu}{k^2 - \mu + i\epsilon} \longleftrightarrow \frac{1}{k^2 - M_0^2 - \Pi(k^2)}, \tag{6}$$

where $\Pi(k^2)$ is a conventional polarization function. It was shown in [11, 12] that the correspondence (6) leads to the definition:

$$\rho(\mu) = \frac{1}{\pi} \frac{Im\Pi(\mu)}{[\mu - M^2(\mu)]^2 + [Im\Pi(\mu)]^2}, \tag{7}$$

where $M^2(\mu) = M_0^2 + Re\Pi(\mu)$. Substitution of the expression (7) into (4) and integration over $\mu$ lead to the results:

$$D_{mn}(q) = i\frac{-g_{mn} + q_m q_n/q^2}{q^2 - M^2(q^2) - iIm\Pi(q^2)} \tag{8}$$

and

$$\hat{G}(q) = i\frac{\hat{q} + q}{q^2 - M^2(q^2) - iq\Sigma(q^2)}. \tag{9}$$

In analogy with these definitions we get the expression for the propagator of vector-spinor unstable field:

$$\hat{G}_{mn}(q) = -\frac{\hat{q} + q}{q^2 - M^2(q^2) - iq\Sigma(q^2)} \{ g_{mn} - \frac{1}{3}\gamma_m \gamma_n - \frac{\gamma_m q_n - \gamma_n q_m}{3q} - \frac{2}{3}\frac{q_m q_n}{q^2} \}. \tag{10}$$

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Note that in Eqs. (9) and (10) we have substituted $q\Sigma(q^2)$ for $\Pi(q^2)$. The expressions (6)–(10) define an effective theory of UP's which follows from the model and the definition (7), that is from the correspondence (6). In this theory the numerators of the expressions (8)–(10) differ from the standard ones. The correspondence between standard and model expressions for the cases of vector (in unitary gauge) and spinor UP is given by the interchange $m \leftrightarrow q$ in the numerators of the standard and model propagators (here $q = \sqrt{q_i q^i}$). As a result the structures of the model propagators lead to the effect of exact factorization, while the standard ones lead to approximate factorization (see the next section).

## 3. Effects of factorization in the processes with UP in an intermediate state

Exact factorization is stipulated by the following properties of the model:
a) smearing the mass shell of UP in accordance with the time-energy UR;
b) specific structure of the numerators of the propagators, Eqs. (8)–(10).
The first factor allows us to describe UP in an intermediate state with the momentum $q$ as a particle in a final or initial states with the variable mass $m^2 = q^2$. In this case, UP is described by the following polarization matrices which differ from the standard on-shell ones by the change $m \to q$ (see also [12]):

$$\sum_{a=1}^{3} e_m^a(\vec{q})\dot{e}_n^a(\vec{q}) = -g_{mn} + \frac{q_m q_n}{q^2} \quad \text{(vector UP)};$$

$$\sum_{a=1}^{2} u_i^{a,\mp}(\vec{q})\bar{u}_k^{a,\pm}(\vec{q}) = \frac{1}{2q^0}(\hat{q} + q)_{ik} \quad \text{(spinor UP)};$$

$$\hat{\Pi}_{mn}(q) = -\frac{1}{4}(\hat{q} + q)\left[g_{mn} - \frac{1}{3}\gamma_m\gamma_n - \frac{\gamma_m q_n - \gamma_n q_m}{3q} - \frac{2}{3}\frac{q_m q_n}{q^2}\right],$$

(vector-spinor UP). $\hspace{6cm}$ (11)

The second factor is the coincidence of the expressions for the propagator numerators (8)–(10) and for the polarization matrices (11). It allows us to represent the amplitude of the process with UP in an intermediate state (see Fig.1) in a partially factorized form:

$$M(p, p', q) = K \sum_a \frac{M_1^{(a)}(p, q) \cdot M_2^{(a)}(p', q)}{P(q^2, M^2)}, \hspace{2cm} (12)$$

where $M^{(a)}$ is a spiral amplitude. The representation (12) is a precondition of factorization, while full exact factorization occurs in the transition probability. The effect of factorization is illustrated in Fig.1 where UP in an intermediate state is signed by crossed line.

Now, we demonstrate the factorization effect in the case of the simplest basic elements of the tree processes, where UP is in the $s-$ channel intermediate state. The vertices are described by the simplest standard Lagrangians for scalar, vector,
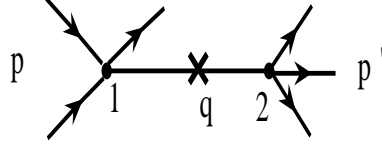
Fig. 1.   Factorization in the reducible diagram.

and spinor particles (see Refs. [21, 22]). Here we add the Lagrangian that describes the interaction of vector-spinor particles with lower-spin ones [26]–[28]:

$$\mathscr{L}_{int} = \frac{f}{m_\pi}\bar{\Psi}^\mu\bar{\Theta}_{\nu\mu}(z,\lambda)N\pi_a^{;\nu} + \text{h.c.}$$
$$+ \frac{g}{2M_N}\bar{\Psi}^\mu\bar{\Theta}_{\nu\mu}(x,\lambda)\gamma^\xi\gamma_5 N\rho_\xi{}^\nu + \text{h.c.}, \tag{13}$$

where

$$\Theta_{\mu\nu}(\lambda,\lambda') = g_{\mu\nu} + \frac{\lambda'-\lambda}{2\,(2\lambda-1)}\gamma_\mu\gamma_\nu\,, \tag{14}$$

$x, z$ are off-shell parameters, and $\lambda$ is the parameter of the Lagrangian of the Rarita-Schwinger free field.

Let us consider the simplest processes which are used further as the basic elements of the method. Note that calculations are made at the "tree level" in the framework of the effective theory, which follows from the model with mass smearing and contains self-energy corrections in the function $\rho(\mu)$. This effective theory is not a gauge one, but the definitions of the model propagators (4) are given in analogy with the standard physical gauge (unitary gauge for massive fields).

The first element is two-particle scattering with UP of any type in the intermediate state.
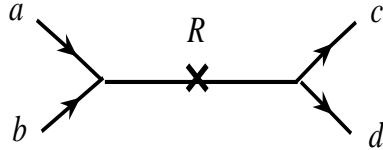


Fig. 2.   Factorization in $2 \to 2$ scattering diagram

By straightforward calculation at the tree level it was shown that the cross-section for all permissible combinations of particles $(a, b, R, c, d)$ can be represented in the universal factorized form [21]:

$$\sigma(ab \to R \to cd) = \frac{16\pi(2J_R+1)}{(2J_a+1)(2J_b+1)\bar{\lambda}^2(m_a,m_b;\sqrt{s})}\frac{\Gamma_R^{ab}(s)\Gamma_R^{cd}(s)}{|P_R(s)|^2}. \tag{15}$$

Here, $s = (p_a+p_b)^2$, $P_R(s)$ is the denominator of the propagator of an unstable particle $R(s)$ which is defined by Eqs. (8) and (9), $\lambda(m_a, m_b; \sqrt{s})$ is normalized

Källén function and $\Gamma_R^{ab}(s) = \Gamma(R(s) \to ab)$ is a partial width of the particle $R(s)$ with variable mass $m = \sqrt{s}$. Note that exact factorization in the process under consideration always takes place for scalar $R$ in both the standard and model treatment. In the case of vector and spinor $R$ the factorization is exact in the framework of the model only. In the standard treatment the expression (15) is valid in the narrow-width approximation (see, for instance, Eq. (37.51) and corresponding comment in [29]). If the diagram depicted in Fig.2 is included into a more complicated one as a sub-diagram, where the particle $c$ and/or $d$ are unstable, then we have to generalize partial width, for instance, $\Gamma_R^{cd(q)}(s, q) = \Gamma(R(s) \to cd(q))$, where $d(q)$ is unstable particle $d$ with mass $m^2 = q^2$ (see the next section). Note that Eq.(15) differs from the corresponding equation (6) in [21] and equation (42) in [22] which contain misprints. Correct expressions can be got in [21, 22] by modification $k_a k_b / k_R \to k_R / k_a k_b$, where $k_p = 2J_p + 1$ and $J_p$ is spin of the particle $p = a, b, R$.

It was shown in Ref.[21] that Eq.(15) is valid for the cases of scalar ($J = 0$), vector ($J = 1$), and spinor ($J = 1/2$) unstable particles. In this work we check by direct calculation that Eq.(15) is also correct in the case of UP with $J = 3/2$, when Eq. (10) is used (for instance, $\Delta$-resonance production). We should note that the expressions which involve vector-spinor UP are valid for the particles on the mass shell in the framework of the standard treatment. However, in the framework of the model, UP is always on its smeared mass shell and these expressions are valid in general case. Moreover, the part of expressions which contains off-shell parameters as well as $\lambda$-parameter disappears in widths and cross-sections. Hence the condition of factorization—the coincidence of the polarization matrix and the numerators of the propagators—is fulfilled in the case under consideration (see Eqs. (10) and (11)).

The second basic element is a three-particle decay with UP in the intermediate state $\Phi \to \phi_1 R \to \phi_1 \phi_2 \phi_3$ (Fig.3), where $R$ is UP of any kind.
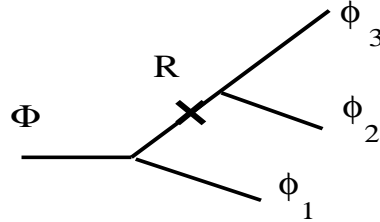


Fig. 3.   Factorization in $1 \to 3$ decay diagram

By straightforward calculations it was shown that the three-particle partial width at the tree level can be represented in the universal factorized form [22]:

$$\Gamma(\Phi \to \phi_1 \phi_2 \phi_3) = \int_{q_1^2}^{q_2^2} \Gamma(\Phi \to \phi_1 R(q)) \frac{q \, \Gamma(R(q) \to \phi_2 \phi_3)}{\pi |P_R(q)|^2} \, dq^2 \,, \qquad (16)$$

where $R$ is a scalar, vector or spinor UP, $q_1 = m_2 + m_3$ and $q_2 = m_\Phi - m_1$. By

direct calculations we also check the validity of the expression (16) for the case of the UP with $J = 3/2$ (see the remark to Eq.(15)). It is seen clearly that the formula (16) can include any factorizable corrections.

By summing over decay channels of $R$, from Eq.(16) we get the well-known convolution formula for the decays with UP in a final state [22]:

$$\Gamma(\Phi \to \phi_1 R) = \int_{q_1^2}^{q_2^2} \Gamma(\Phi \to \phi_1 R(q)) \, \rho_R(q) \, dq^2 \, . \tag{17}$$

In Eq.(17) smearing the mass of unstable state $R$ is described by the probability density $\rho_R(q)$:

$$\rho_R(q) = \frac{q \, \Gamma_R^{tot}(q)}{\pi \, |P_R(q)|^2}. \tag{18}$$

The expression (18) is connected with Eq.(7) by the relation $Im\Pi(q) = q \, \Gamma_R^{tot}(q)$.

The factorized expressions (16) and (17) are applied successfully for the description of the decay $B \to \rho D$ [12], decay properties of $\phi(1020)$-meson [12] and $t$-quark [30, 31, 32], lightest chargino and next-to-lightest neutralino [33]. Moreover, the formula (16) describes the decays $\mu \to e\bar{\nu}_e\nu_\mu$, $\tau \to e\bar{\nu}_e\nu_\tau$, and $\tau^- \to \nu_\tau\pi^-\pi^0$ with great accuracy (see the next section). It should be noted that, in analogy with two-particle scattering, the factorization in the expression for the width (16) is also exact within the framework of the model and approximate in the standard treatment (convolution method). Besides, we note that the expressions (15) and (16) significantly simplify calculations in comparison with the standard ones.

## 4. Factorization method in the model of UP's with smeared mass

The method is based on exact factorization of the simplest processes with UP in an intermediate state that were considered in Section 3. The factorization method has applicability to such Feynman diagrams that can be disconnected into two components by cutting some line corresponding to timelike momentum transfer. For instance, it is applicable to the complicated scattering and decay-chain processes which can be reduced to a chain of the basic elements (15) and (16). Next, we consider some examples of such processes.

$$1) \, a + b \to R_1 \to c + R_2 \to c + d + f \text{ (Fig.4)}. \tag{19}$$

The cross-section of this process is a combination of the expressions (15) and (16):

$$\sigma(ab \to R_1 \to cdf) =$$

$$\frac{16k_{R_1}}{k_a k_b \bar{\lambda}^2(m_a, m_b; \sqrt{s})} \frac{\Gamma_{R_1}^{ab}(s)}{|P_{R_1}(s)|^2} \int_{q_1^2}^{q_2^2} \Gamma(R_1(s) \to cR_2(q)) \frac{q \, \Gamma_{R_2}^{df}(q)}{|P_{R_2}(q)|^2} \, dq^2, \tag{20}$$
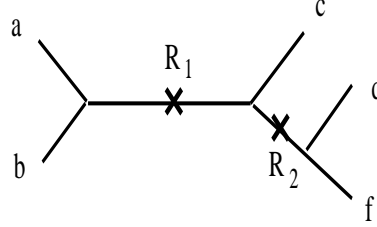
Fig. 4.    Factorization in $2 \to 3$ scattering-decay diagram

where $k_p = 2J_p + 1$. It should be noted that the factorization effectively reduces the number of independent kinematic variables which have to be integrated. In the standard approach for the process $2 \to 3$ the number of the variables, which uniquely specify a point in the phase space, in the general case is $N = 3n - 4 = 5$, from which four variables have to be integrated [34]. Some of this variables can be integrated out if a specific symmetry of the process occur. In the framework of the approach suggested the number of integrated variables is always $N_M = 1$. The same effect of variable reduction takes place for the case of the basic processes of scattering and decay which were considered in the previous section.

The expression (20) can be used for fast evaluation of cross sections of some scattering processes both in cosmology and collider physics. For instance, it is valid for the description of the annihilation process with the lightest supersymmetric particle in a final state [33].

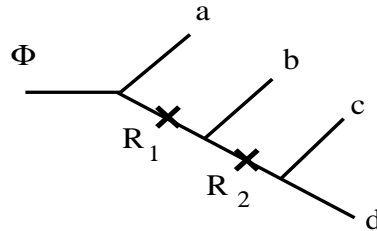$$2) \, \Phi \to a + R_1 \to a + b + R_2 \to a + b + c + d \text{ (Fig.5).} \tag{21}$$



Fig. 5.    Factorization in $1 \to 4$ decay diagram

The width of this decay-chain process is given by doubling the formula (16):

$$\Gamma(\Phi \to abcd) = \frac{1}{\pi^2} \int_{q_1^2}^{q_2^2} \frac{q\,\Gamma(\Phi \to aR_1(q))}{|P_{R_1}(q)|^2} \times$$

$$\times \int_{g_1^2}^{g_2^2} \Gamma(R_1(q) \to bR_2(g)) \frac{g\,\Gamma(R_2(g) \to cd)}{|P_{R_2}(g)|^2} \, dg^2 \, dq^2. \qquad (22)$$

Note that in the general case of $n$-particle decay the number of kinematic variables, which uniquely specify a point in the phase space, is $N = 3n - 7 = 5$ [34], while the method gives $N_M = 2$ (see comment to the previous case). Thus, we have a significant simplification of calculations. Some examples of processes which can be described by the compact formula (22) are considered in [32], where the relation between convolution method and decay-chain method is analyzed.

$$3)\, a + b \to c + R \to c + d + e \; (\text{Fig.6}). \qquad (23)$$



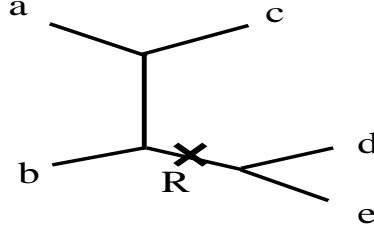Fig. 6.   Factorization in $a + b \to c + R \to c + d + e$ process.

The cross-section of this $t$-channel process is described by convolution of the cross-section $\sigma(ab \to cR)$ and the width $\Gamma(R \to de)$:

$$\sigma(ab \to cde) = \frac{1}{\pi} \int_{q_1^2}^{q_2^2} \sigma(ab \to cR(q)) \frac{q\Gamma(R(q) \to de)}{|P_R(g)|^2} \, dq^2. \qquad (24)$$

This formula can be applied to the description of the processes $e^+e^- \to \gamma Z \to \gamma f\bar{f}$ and $eN \to e\Delta \to e\pi N$. The diagram in Fig.6 illustrates a class of processes at the tree level with fermion-antifermion pair in the one-pole approximation, i.e. generated from the decay of $R$ only. However, the formula (24) can be easily generalized taking account of factorizable radiative corrections. For instance, such a generalization of the expression (24) was made in [35] for the description of the process $e^+e^- \to \gamma Z \to \gamma \sum_f \nu_f \bar{\nu}_f$, $f = e, \mu, \tau$, with $\nu\bar{\nu}$-pairs being produced by $Z$-decay only ("single-pole" resonant production). Note that additional non-resonant "ladder" diagrams also contribute to the process $e^+e^- \to \gamma \nu\bar{\nu}$. The resonant events can be still separated in certain kinematic regions [36]. In our calculations [35] (with the kinematic cut corresponding to the event selection of Ref.[36]) we have taken into

account ISR and principal part of radiative corrections. These corrections do not change the structure of the expression (24) and satisfy the condition of factorization. The results are in good agreement with the experimental data and SM predictions at $\sqrt{s} = 185 - 210\,\text{GeV}$ [35]. We should note, however, that our calculations are valid in the energy domain, where the selection of the resonant events is possible.

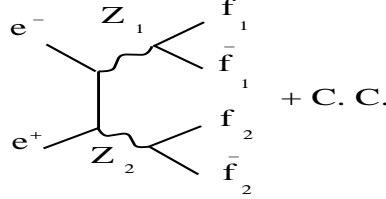$$4)\, e^+e^- \to ZZ \to \sum_{i,k} \bar{f}_i f_i \bar{f}_k f_k \ (\text{Fig.7}). \tag{25}$$



Fig. 7.    $Z$-pair production process.

Now, we consider the process of $Z$-pair production (or four-fermion production in the double-pole approximation). Direct application of the model to the process $e^+e^- \to ZZ$ or using the factorization method for the full process $e^+e^- \to ZZ \to \sum_{i,k} \bar{f}_i f_i \bar{f}_k f_k$ (double-pole approach) gives the following expression for cross-section at the tree level [18]:

$$\sigma^{tr}(e^+e^- \to ZZ) = \int\int \sigma^{tr}(e^+e^- \to Z_1(m_1)Z_2(m_2))\, \rho_Z(m_1)\, \rho_Z(m_2)\, dm_1\, dm_2, \tag{26}$$

where $\sigma^{tr}(e^+e^- \to Z_1(m_1)Z_2(m_2))$ is defined in a standard way for the case of fixed boson masses $m_1$ and $m_2$, and probability density of mass $\rho(m)$ is defined by the expression:

$$\rho_Z(m) = \frac{1}{\pi}\, \frac{m\,\Gamma_Z^{tot}(m)}{(m^2 - M_Z^2)^2 + (m\,\Gamma_Z^{tot}(m))^2}. \tag{27}$$

Similar expressions can be written for the processes $e^+e^- \to W^+W^-$ [19] and $e^+e^- \to ZH$ [35]. To describe exclusive processes, such as $e^+e^- \to ZZ \to f_i\bar{f}_i f_k\bar{f}_k$, one has to substitute partial $q$-dependent width $\Gamma_Z^i(q) = \Gamma(Z(q) \to f_i\bar{f}_i)$ into the expression (27) instead of the total width $\Gamma_Z^{tot}(m)$. Note that the expression similar to (24) can be written in the standard approach as a result of integration over the phase space variables which describe $4f$-states in the semi-analytical approximation (SAA) [37]. Within the framework of the model considered, formula (24) is derived exactly without any approximations. Note also that as a rule the deviation of the standard exact results from the model ones is negligible (see Eq. (32)).

The processes $e^+e^- \to ZZ, W^+W^-, \gamma Z, ZH$ were considered in detail [17, 18, 35] taking account of the relevant radiative corrections. It was shown that the results of the model calculations are in good agreement with the experimental LEP II data

and coincide with standard Monte-Carlo results with great accuracy. At the same time the factorization method significantly simplifies the calculation procedures in comparison with the standard ones that should consider about ten thousands diagrams (see, for example, [38] and comments in [18]).

Using the factorization method one can describe complicated decay-chain and scattering processes in a simple way. The same results can occur within the frame of standard treatment as the approximations. Such approximations are known as narrow-width approximation (NWA) [39, 40], convolution method (CM) [30]–[32], decay-chain method (DCM) [32] and semi-analytical approach (SAA) [37]. All these approximations get a strict analytical formulation within the framework of the factorization method. For instance, NWA includes five assumptions which was considered in detail in [39]. The factorization method contains just one assumption— non-factorizable corrections are small (the fifth assumption of NWA). The method suggested can be applied to very complicated decay-chain and scattering processes by combining the expressions considered above. In this case we have not a strict and general standard analog of such approximation.

Now, we consider some ways of evaluation of the method error which we define as the deviation of the model results from the strict standard ones. For a scalar UP the error always equals zero in accordance with the definition (6). For a vector UP the error is caused by the following difference:

$$\delta\eta_{\mu\nu} = \eta_{\mu\nu}(q^2) - \eta_{\mu\nu}(m^2) = q_\mu q_\nu \frac{m^2 - q^2}{m^2 q^2}, \tag{28}$$

where $\eta_{\mu\nu}(m^2)$ and $\eta_{\mu\nu}(q^2)$ are standard and model numerators of vector propagators in the physical gauge. In the case of meson-pair production $e^+e^- \to \rho^0, \omega, ... \to \pi^+\pi^-, K^+K^-, \rho^+\rho^-, ...$ the deviation equals zero too, due to vanishing contribution of the transverse parts of the amplitudes in both cases:

$$M^{trans}(q) \sim \bar{e}^-(p_1)\hat{q}e^-(p_2) = \bar{e}^-(p_1)(\hat{p}_1 + \hat{p}_2)e^-(p_2) = 0. \tag{29}$$

In the case of the high-energy collisions $e^+e^- \to Z \to f\bar{f}$ (we neglect $\gamma - Z$ interference) the transverse part of the amplitude is:

$$M^{trans}(q) \sim \bar{e}^-(p_1)\hat{q}(c_e - \gamma_5)e^-(p_2) \cdot \bar{f}^+(k_1)(c_f - \gamma_5)f^+(k_2) \tag{30}$$

and we get at $q^2 \approx M_Z^2$:

$$\delta M \sim \frac{m_e m_f}{M_Z^2} \frac{M_Z - q}{M_Z}. \tag{31}$$

Thus, an error of the factorization method at the vicinity of resonance is always small, moreover, it is suppressed by small factor $m_e m_f / M_Z^2$. The similar estimations can be easily done for the case of a spinor UP.

The relative deviation of the model cross section of the boson-pair production with consequent decay of the bosons to fermion pairs is [18]:

$$\epsilon_f \sim 4\frac{m_f}{M}[1 - M \int_{m_f^2}^{s} \frac{\rho(q^2)}{q} \, dq^2], \tag{32}$$

where $M$ is a boson mass. For the case $f = \tau$ a deviation is maximal, $\epsilon_\tau \sim 10^{-3}$. It should be noted that the deviations that are caused by the approach at the tree level are significantly smaller then the errors caused by the uncertainty in taking account of radiative corrections [19]. Thus, the the error of the method at the tree level in the case of vector UP is, as a rule, negligible.

By straightforward calculations we evaluate the relative deviations of the model partial width from standard one, that is, $\epsilon = (\Gamma^M - \Gamma^{st})/\Gamma_M$ for the case of $\mu$ and $\tau$ decays. We get:

$$\epsilon(\mu \to e\nu\bar{\nu}) \approx 5 \cdot 10^{-4}; \ \ \epsilon(\tau \to e\nu\bar{\nu}) \approx 3 \cdot 10^{-6}; \ \ \epsilon(\tau \to \mu\nu\bar{\nu}) \approx 3 \cdot 10^{-2}. \tag{33}$$

The deviation is suppressed by the factor $k = m_{l1}^2/m_{l2}^2$, which is small in the first and second case and large in the last case. In the case of the decay $\tau^- \to \nu_\tau \pi^- \pi^0$, suppression factor is very small, $k = (m_{\pi_0}^2 - m_{\pi_-}^2)^2/m_\tau^4 \sim 10^{-7}$.

In the case of a spinor UP in an intermediate state a deviation is of the order of $(M_f - q)/M_f$. It can be large when $q$ is far from the resonance region. However, in analogy with vector UP, this deviation can be suppressed by small factor too, and we have to control this effect in every case under consideration. The same effect can occur in the case of the vector-spinor UP.

## 5. Conclusion

The model of UP's leads to effective theory of UP's with a specific structure of vector, spinor and vector-spinor propagators. Such a structure gives rise to the effects of exact factorization in a broad class of the processes with UP's in intermediate states. These effects allow us to develop the factorization method for the description of the complicated processes with participation of an arbitrary type of UP's. The method suggested is simple and convenient tool for deriving the formulae for cross sections and decay rates in the case of complicated scattering and decay-chain processes. The factorization method can be used as some analytical analog of NWA, which enables us to evaluate the error of the approach at the tree level in a simple way. We have shown that these errors, as a rule, are significantly smaller then the ones caused by the uncertainty in taking account of radiative corrections. It should be noted that the applicability of the method is limited to the energy scales where the non-resonant or non-factorizable contributions can be neglected.

The factorization method based on the model of UP with a smeared mass can be treated in two various ways. On the one hand, it follows from the specific structure of propagators and can be interpreted as some heuristic (irrespective of the model) way to evaluate decay rates and cross sections easily with the help of the concise and convenient expressions. On the other hand, the model, from which the factorization method follows, is based on the fundamental properties of UP—time-energy UR (i.e., smearing the mass). Thus, the method can be also used as some physical basis for development of precision tools of rapid and easy calculations.

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